Simple Manipulation Rules for Expected Value and Variance

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October 5, 2021

X: Random Continuous Variable
Definition of Expected Value and Variance:

$$E[X] = \int XP(X)dX$$
$$Var[X] = E[(X - E[X])^2] = \langle (X - \langle X \rangle)^2 \rangle$$

Expected Value rules for one random variable:

$$\langle f(X) + b \rangle = \langle f(X) \rangle + b$$
$$\langle af(X) \rangle = a\langle f(X) \rangle$$
$$\langle c \rangle = c$$

$$\langle f(X) + g(X) \rangle = \langle f(X) \rangle + \langle g(X) \rangle$$
$$\langle fg \rangle = ugghhhh$$

Variance rules for one random variable:

$$Var(f(X) + b) = Var(f(X))$$
$$Var(af(X)) = a^2 Var(f(X))$$
$$Var(c) = 0$$

$$Var(f + g) = Var(f) + Var(g) + 2(\langle fg \rangle - \langle f \rangle \langle g \rangle)$$

Expected Value rules for multiple, mutually independent variables:

$$E[Y] = \int Y \prod P_i(X_i) dX_i$$

$$Y = f(X_1, X_2, ..., X_n) + g(X_1, X_2, ..., X_n)$$
$$\langle Y \rangle = \langle f(X_1, X_2, ..., X_n) \rangle + \langle g(X_1, X_2, ..., X_n) \rangle$$

$$Y = f_1(X_1) \cdot f_2(X_2) \cdot ... \cdot f_n(X_n)$$
$$\langle Y \rangle = \langle f_1(X_1) \rangle \cdot \langle f_2(X_2) \rangle \cdot ... \cdot \langle f_n(X_n) \rangle$$

$$\langle af(X_1, X_2, ..., X_n) \rangle = a\langle f(X_1, X_2, ..., X_n) \rangle$$
$$\langle f(X_1, X_2, ..., X_n) + b \rangle = \langle f(X_1, X_2, ..., X_n) \rangle + b$$
$$\langle c \rangle = c$$

Variance rules for multiple, mutually independent variables:

$$Var[Y] = \langle (Y - \langle Y \rangle)^2 \rangle = \langle Y^2 \rangle - \langle Y \rangle^2$$

$$Var[f + g] = Var[f] + Var[g] - 2(\langle fg \rangle - \langle f \rangle \langle g \rangle)$$
$$Var[fg] = \langle f^2 g^2 \rangle - \langle fg \rangle^2$$

$$Var[af(X_i)] = a^2 Var[f(X_i)]$$
$$Var[f(X_i) + b] = Var[f(X_i)]$$
$$Var[c] = 0$$